37. Filters and Filter Types

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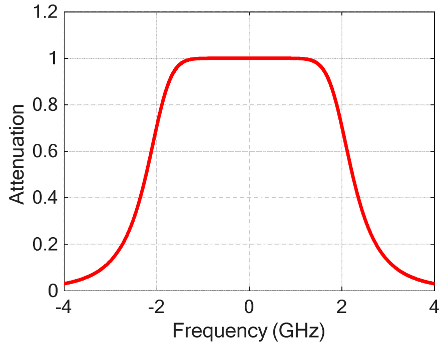


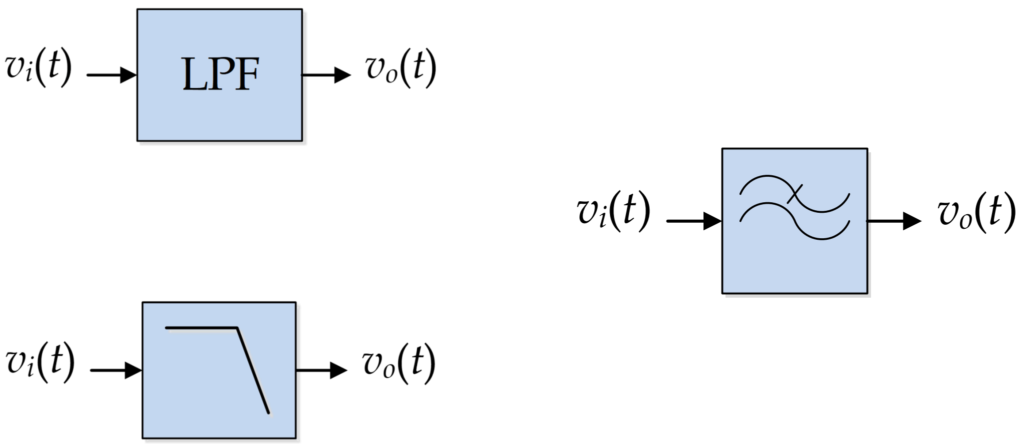
A common type of system is a **Filter**

Filters pass certain frequencies and reject others. In other words, a sinusoid of certain frequency might pass through the filter while another (with different frequency) might not. Similar to how an oil filter in your car blocks suspended particles from passing along with the oil.

There are four main filter types that we frequently use and design:

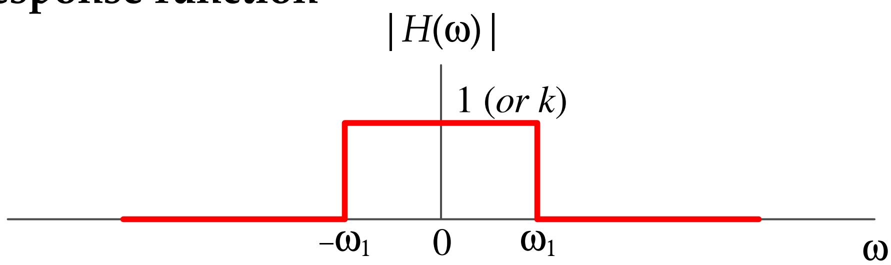
- **Low-Pass Filter (LPF)**
- **High-Pass Filter (HPF)**
- **Band-Pass Filter (BPF)**
- **Band-Stop Filter (or Notch Filter)**

Low-Pass Filter (LPF): Passes low frequencies and rejects high frequencies. It has different symbols:

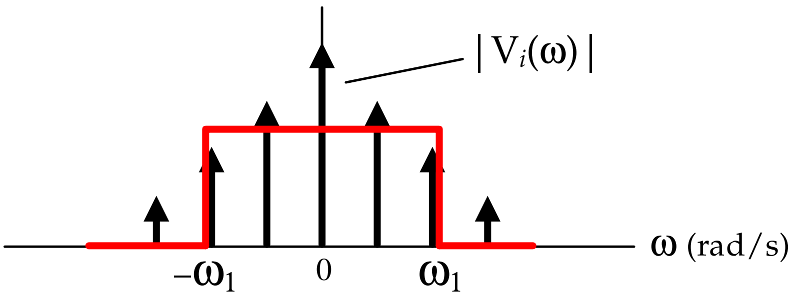


Ideal LPF: Frequency response function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

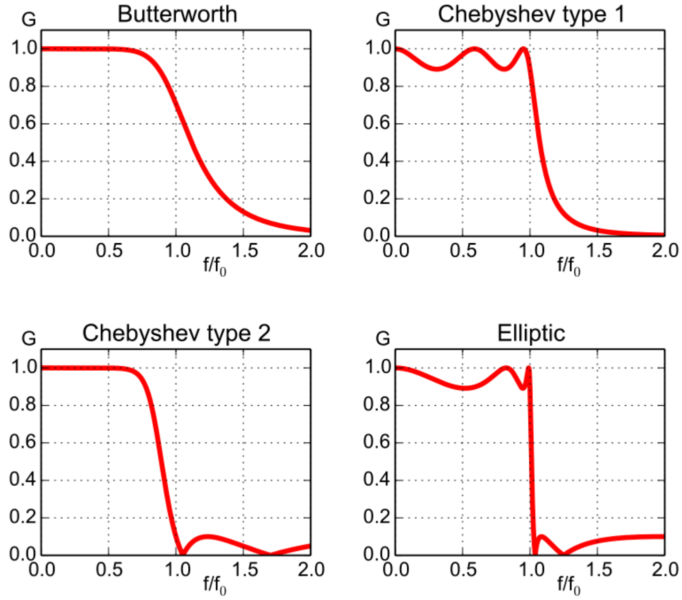


$$|V_o(\omega)| = |H(\omega)| \times |V_i(\omega)|$$

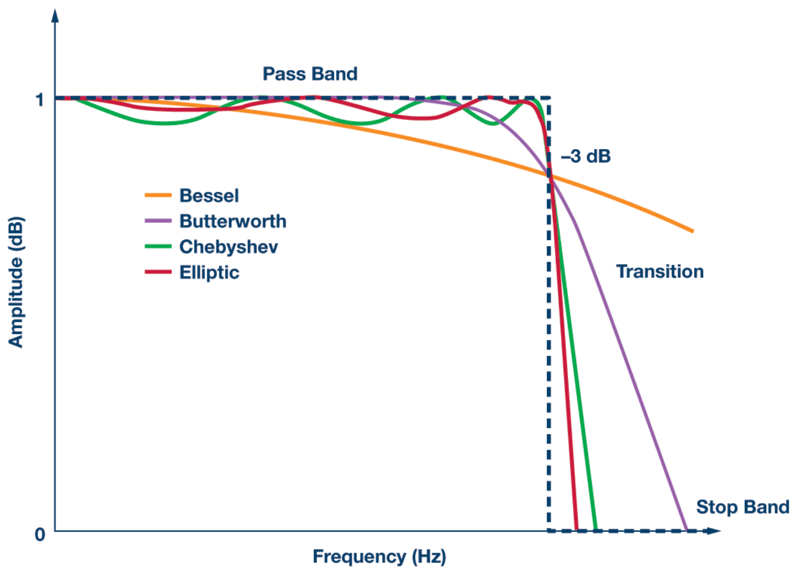


Cut-off frequency is ω_1

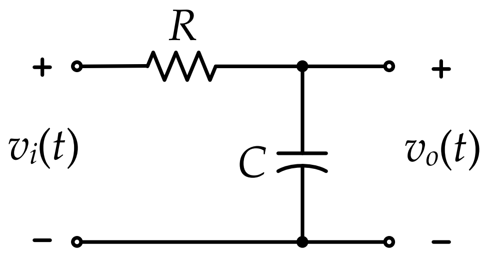
Practical LPF: Frequency response function



Practical cut-off frequency is ω_1 where $H(\omega_1) = V_o(\omega_1)/V_i(\omega_1) = H(\omega)_{max}/\sqrt{2} = 0.7071\ H(\omega)_{max}$ (half-power point, or -3 dB point).



Example LPF circuit: Firs-order RC circuit



$$\omega_1 = \frac{1}{RC} \ \text{[rad/s]}$$

Q1. Design a LPF to pass a signal $x(t)$ with bandwidth $B_x = 100$ Hz?

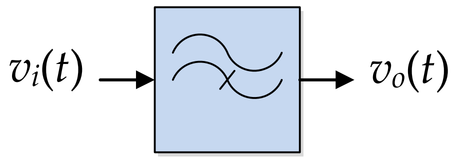
Q1. Solution. To avoid rejecting any of the significant harmonics of the signal $x(t)$, the LPF cut-off frequency has to be equal or greater than the signal bandwidth. Hence,

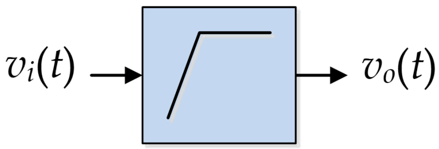
$$\omega_1 \geq W_x$$

$$\omega_1 \geq 2\pi B_x = 2\pi \times 100 = 200\pi \ \text{[rad/s]}$$

Use the above RC circuit with $C = 10$ μF and $R = 159$ Ω.

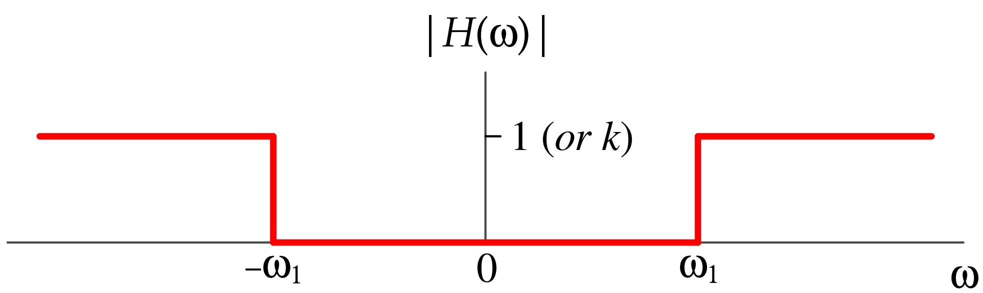
High-Pass Filter (HPF): Passes high frequencies and rejects low frequencies. It has different symbols:



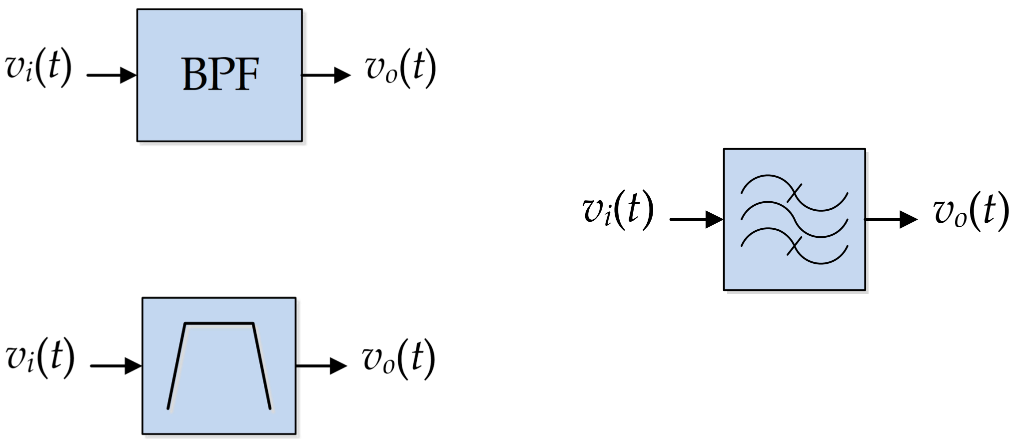


Ideal HPF: Frequency response function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

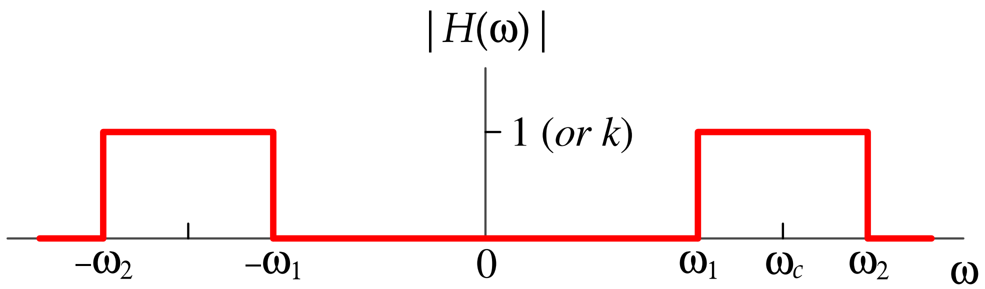


Band-Pass Filter (BPF): Rejects low and high frequencies but passes frequencies within a certain frequency band. It has different symbols:



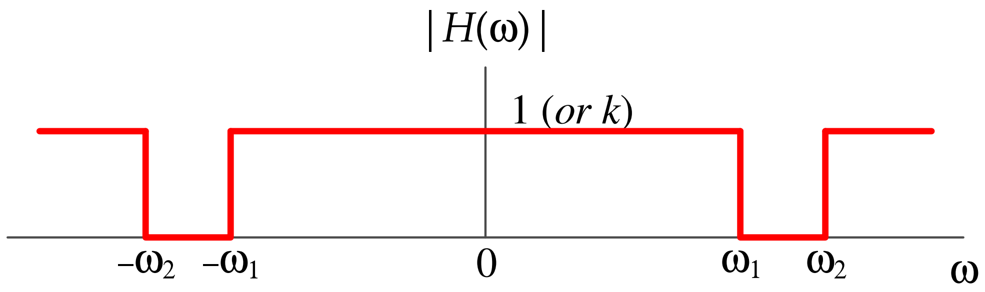
Ideal BPF: Frequency response function

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$



Band-Stop Filter (or Notch Filter): Passes all frequencies but rejects a certain frequency band.

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$



Q2. A filter has an impulse response function $h(t) = 10\,\mathrm{sinc}(10t)$. Identify the type of the filter and determine its cut-off frequency.

Q2. Solution. Filters can be easily identified in frequency-domain by looking at $H(\omega)$, so we sketch the frequency response function,

$$H(\omega) = \mathcal{F}\{h(t)\} = \mathcal{F}\{10\,\mathrm{sinc}(10t)\}$$

Using Fourier transform duality property,

$$\mathrm{rect}(t) \Leftrightarrow \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$\mathrm{sinc}\left(\frac{t}{2\pi}\right) \Leftrightarrow 2\pi\ \mathrm{rect}(-\omega)$$

$$\text{sinc}\left(\frac{t}{2\pi}\right) \Leftrightarrow 2\pi\ \text{rect}(-\omega)$$

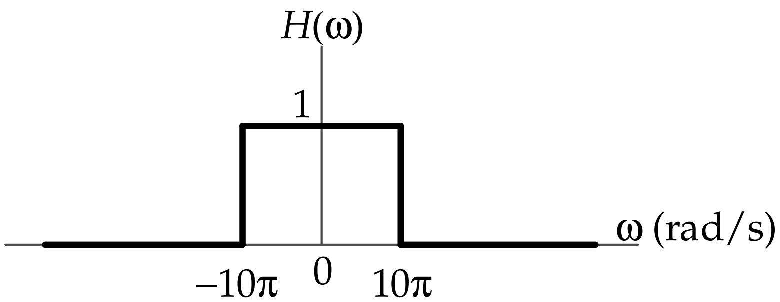
$$\text{sinc}\left(\frac{20\pi\ t}{2\pi}\right) \Leftrightarrow \frac{2\pi}{20\pi}\ \text{rect}\left(\frac{\omega}{20\pi}\right)$$

$$\text{sinc}(10t) \Leftrightarrow \frac{1}{10}\ \text{rect}\left(\frac{\omega}{20\pi}\right)$$

$$10\,\text{sinc}(10t) \Leftrightarrow \frac{10}{10}\ \text{rect}\left(\frac{\omega}{20\pi}\right)$$

$$H(\omega) = \mathcal{F}\{10\,\text{sinc}(10t)\} = \text{rect}\left(\frac{\omega}{20\pi}\right)$$

Sketching $H(\omega)$ we can easily see that the filter is an **ideal LPF**.

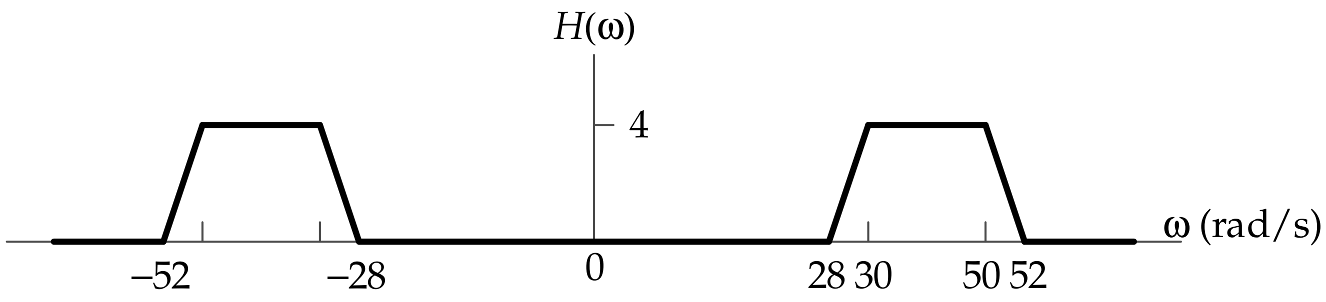


And the cut-off frequency of the filter is,

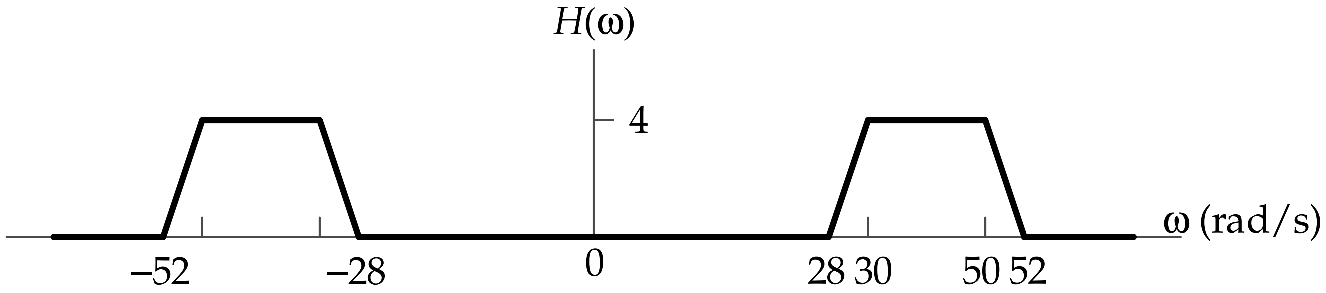
$$\omega_1 = 10\pi\ [\text{rad/s}]$$

$$f_1 = \frac{10\pi}{2\pi} = 5\ [\text{Hz}]$$

Q3. A filter has the following frequency response function $H(\omega)$. Identify the type of the filter and determine its cut-off frequencies.



Q3. Solution. This is a practical BPF (notice the non-abrupt roll-off). Cut-off frequencies are at ω_1 and ω_2 where $H(\omega_1) = H(\omega_2) = 0.7071\ H(\omega)_{max} = 0.7071 \times 4 = 2.8284$ (half-power point).



Writing the linear equation part of $H(\omega)$, where slope is $m = 4/2 = 2$,

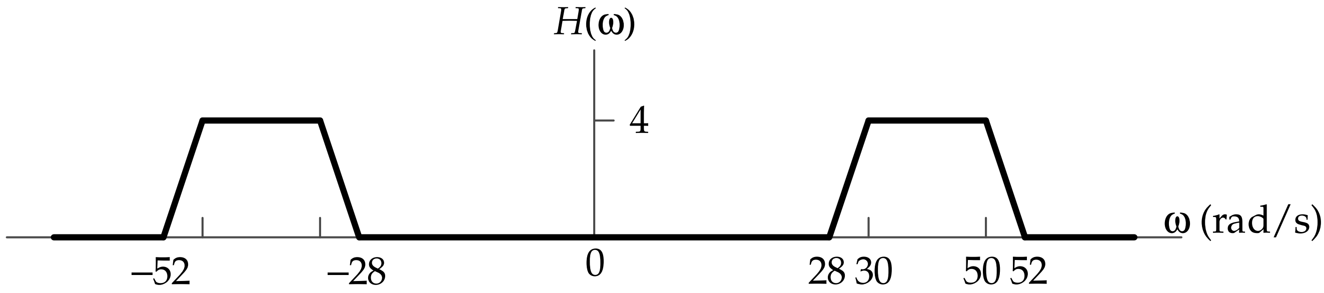
$$H(\omega) = m\,\omega + b = 2\omega + b, \quad \omega \in [28, 30]$$

Substituting the value $H(\omega) = 0$ when $\omega = 28$,

$$0 = 2 \times 28 + b$$

gives $b = -56$. Hence,

$$H(\omega) = 2\omega - 56, \quad \omega \in [28, 30]$$



In order to get

$$H(\omega_1) = 2\omega_1 - 56 = 0.7071 \times 4 = 2.8284$$

The cut-off frequency is

$$\omega_1 = 29.4142 \text{ [rad/s]}$$

Similarly,

$$\omega_2 = 50.5858 \text{ [rad/s]}$$

Notice that the negative-frequency part has even symmetry with the positive-frequency part.